Question 6 (8 marks)

Let .

(a) Determine the three cube roots of . (3 marks)

(b) Consider the polynomial , where is a real constant.  
  
Given that , solve the equation . (5 marks)

Question 6 (8 marks)

Let .

(a) Determine the three cube roots of . (3 marks)

|  |
| --- |
| Solution |
| Hence the cube roots are: |
| Specific behaviours |
| ✓ writes in polar form  ü obtains one correct root  ü correctly states all roots |

(b) Consider the polynomial , where is a real constant.  
  
Given that , solve the equation . (5 marks)

|  |
| --- |
| Solution |
| has real coefficients and so and are factors:  Hence , where is a real constant.  Comparing coefficients of then .  Zeros of second quadratic factor:  Solutions are .  *Note that it is possible to deduce , but this is not required.* |
| Specific behaviours |
| ✓ indicates or is a factor of  ü correctly determines quadratic factor of  ✓ determines second quadratic factor  ü obtains a zero from second quadratic factor  ü states all solutions |

Question 5 (6 marks)

Let , where and are real constants.

One of the roots of is .

(a) Determine the value of the constant and the value of the constant . (4 marks)

(b) Show all the roots of in the complex plane below. (2 marks)

<EFOFEX>
id:fxd{f8f02ac7-86a6-4f09-8152-b64f5b47a34a}

FXData:

</EFOFEX>

Question 5 (6 marks)

Let , where and are real constants.

One of the roots of is .

(a) Determine the value of the constant and the value of the constant . (4 marks)

|  |
| --- |
| Solution |
| Product of factors using given root and conjugate:  Hence  Comparing coefficients:  Hence from expansion of factors, and . |
| Specific behaviours |
| ✓ indicates conjugate is another root  ü obtains quadratic factor of  ü uses quadratic factor to obtain linear factor  ü states correct values for and |

(b) Show all the roots of in the complex plane below. (2 marks)

<EFOFEX>
id:fxd{86550580-e8b8-42eb-b4a5-a2c25d6a3df9}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram |
| Specific behaviours |
| ✓ shows given root and conjugate  ü shows real root |

Question 4 (6 marks)

Consider , where is a real constant.

The equation has a solution .

(a) State a second solution to . (1 mark)

(b) Deduce that . (3 marks)

(c) Determine all other solutions of the equation . (2 marks)

Question 4 (6 marks)

Consider , where is a real constant.

The equation has a solution .

(a) State a second solution to . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates conjugate root |

(b) Deduce that . (3 marks)

|  |
| --- |
| Solution |
| Consider constant term:  Consider coefficient:  Hence coefficient . |
| Specific behaviours |
| ü factors into known and unknown quadratics  ü deduces constant term in quadratic  ü deduces coefficient in quadratic and hence value of |

(c) Determine all other solutions of the equation . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ factors quadratic  ü states both solutions |

Question 8 (7 marks)

(a) Solve the equation , where . (2 marks)

(b) and are the three cube roots of unity.

(i) State the value of and the value of . (1 mark)

Let and .

(ii) Show that and . (4 marks)

Question 8 (7 marks)

(a) Solve the equation , where . (2 marks)

|  |
| --- |
| Solution |
| Hence |
| Specific behaviours |
| ✓ applies De Moivre's theorem  ü correct root |

(b) and are the three cube roots of unity.

(i) State the value of and the value of . (1 mark)

|  |
| --- |
| Solution |
| Sum of roots:  Product of roots: |
| Specific behaviours |
| ✓ sum and product |

Let and .

(ii) Show that and . (4 marks)

|  |
| --- |
| Solution |
| Hence  And |
| Specific behaviours |
| ✓ simplifies  ü simplifies  ü derives value for sum  ü derives value for product |

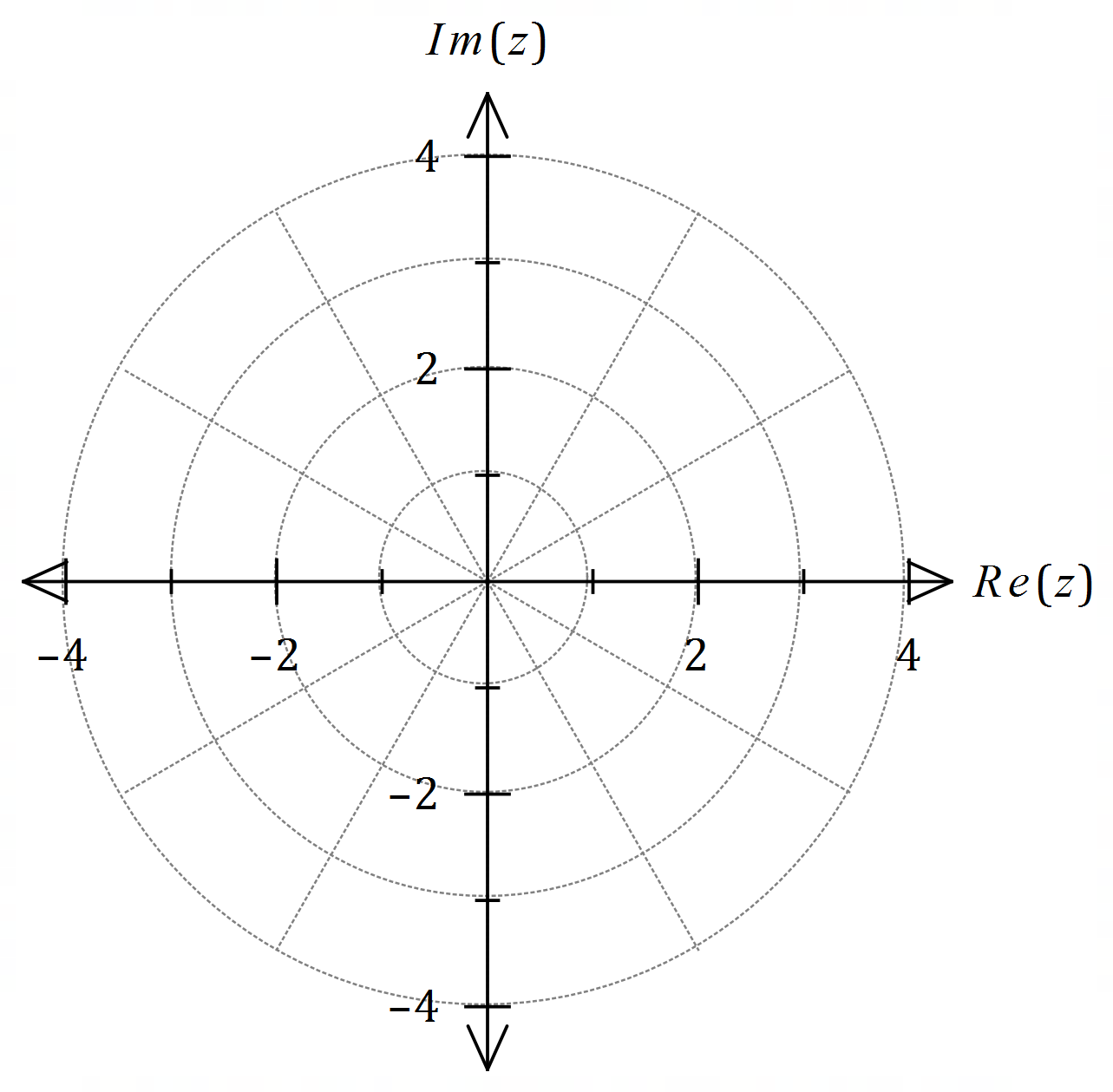
Question 6 (5 marks)

Consider the equation .

One root of the equation is .

(a) Solve the equation, giving all solutions in Cartesian form. (4 marks)

(b) Locate all the roots of the equation on the Argand diagram below. (1 mark)



Question 6 (5 marks)

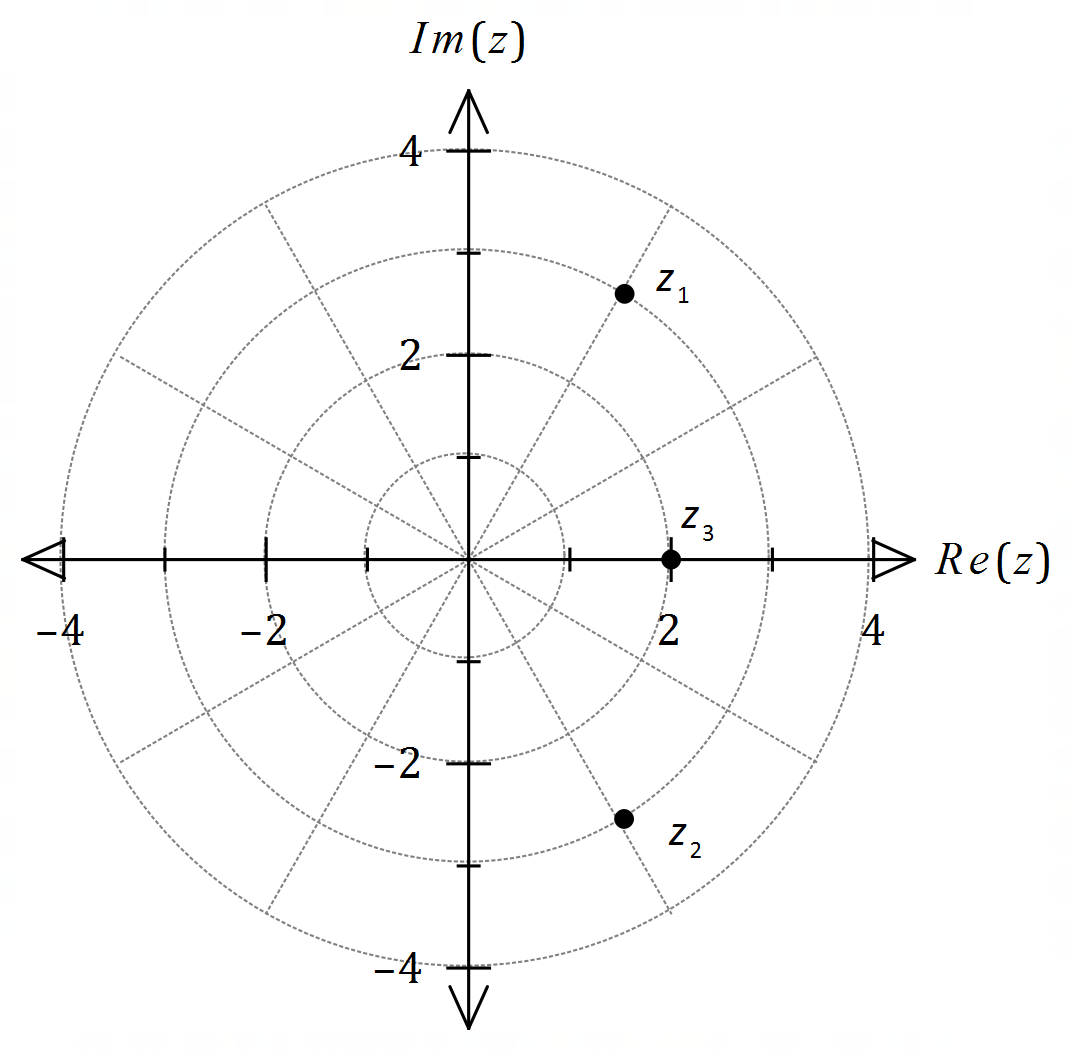
Consider the equation .

One root of the equation is .

(a) Solve the equation, giving all solutions in Cartesian form. (4 marks)

|  |
| --- |
| **Solution** |
| Equation has real coefficients - complex roots will occur as conjugate pairs:  Since the equation must equal then  But and so  Hence solutions are . |
| **Specific behaviours** |
| ✓ second root in polar form   expresses both polar roots in Cartesian form   uses product of roots and term   third solution |

(b) Locate all the roots of the equation on the Argand diagram below. (1 mark)



|  |
| --- |
| **Solution** |
| See diagram |
| **Specific behaviours** |
| ✓ correct locations |

Question 2 (6 marks)

Let , and .

(a) Determine the modulus and argument of . (4 marks)

(b) Determine the smallest positive integer such that is imaginary. (2 marks)

Question 2 (6 marks)

Let , and .

(a) Determine the modulus and argument of . (4 marks)

|  |
| --- |
| **Solution** |
| and |
| **Specific behaviours** |
| ✓ expresses and in polar form   indicates powers of and   forms product   states modulus and argument |

(b) Determine the smallest positive integer such that is imaginary. (2 marks)

|  |
| --- |
| **Solution** |
| If is imaginary, then must be even multiple of . Hence . |
| **Specific behaviours** |
| ✓ indicates argument restriction   correct value of |

Question 4 (6 marks)

Let where .

(a) Clearly show that is a factor of . (2 marks)

(b) Solve the equation . (4 marks)

Question 4 (6 marks)

Let where .

(a) Clearly show that is a factor of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes correctly   clearly shows terms sum to |

(b) Solve the equation . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of conjugate for second factor   factorises   obtains third solution   lists all four solutions |

Question 1 (4 marks)

Consider the equation .

(a) Show that is a solution of the equation. (1 mark)

(b) Determine other solutions of the equation. (3 marks)

Question 1 (4 marks)

Consider the equation .

(a) Show that is a solution of the equation. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ fully expands each term |

(b) Determine other solutions of the equation. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ factors cubic   expression for   both solutions, simplified |

Question 5 (9 marks)

Let .

(a) Determine the real constants and , where . (2 marks)

(b) By first expressing and in polar form, write in polar form. (3 marks)

(c) Hence determine an exact value for . (2 marks)

(d) Determine in Cartesian form. (2 marks)

Question 5 (9 marks)

Let .

(a) Determine the real constants and , where . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ rationalises   states values |

(b) By first expressing and in polar form, write in polar form. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses terms in polar form   modulus of   argument of |

(c) Hence determine an exact value for . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equates real parts   states exact value |

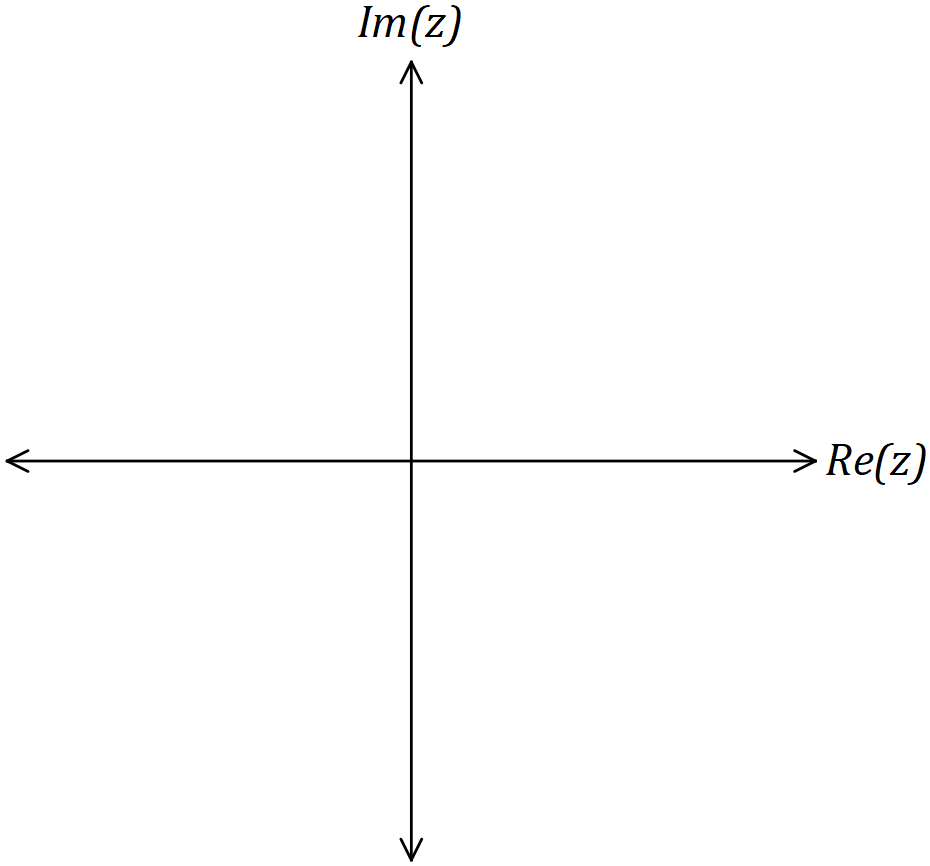
(d) Determine in Cartesian form. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ applies de Moivre's Theorem   correct value |

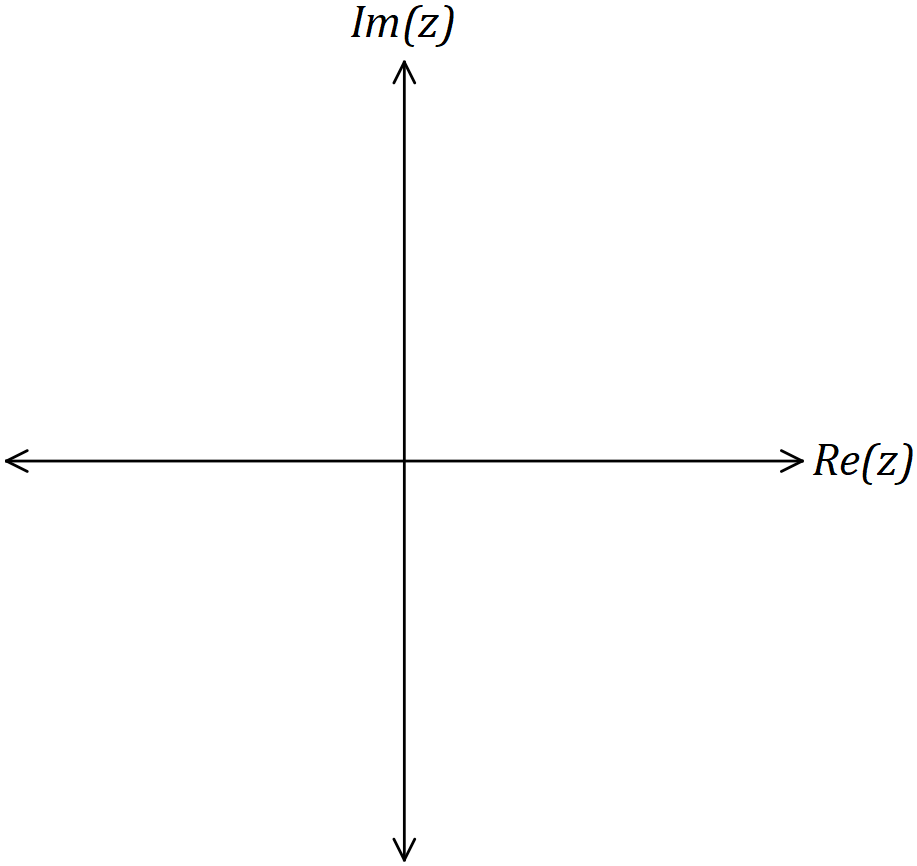
Question 7 (7 marks)

On the Argand planes below, sketch the locus of the complex number given by the following.

(a) . (3 marks)



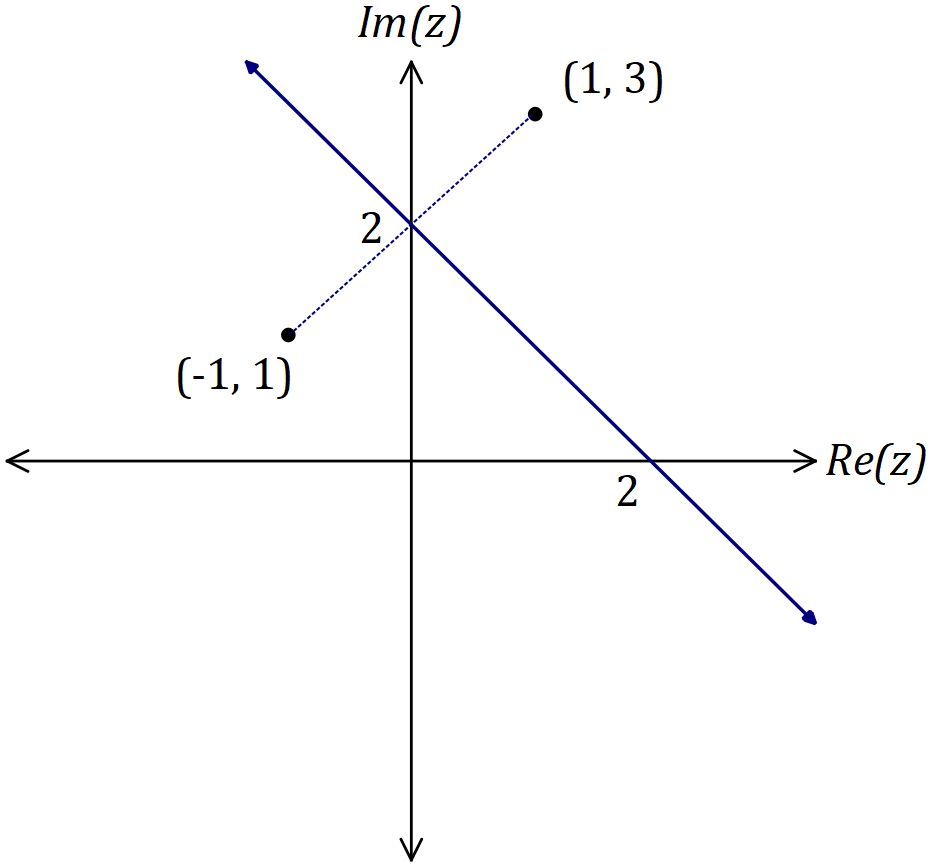
(b) . (4 marks)



Question 7 (7 marks)

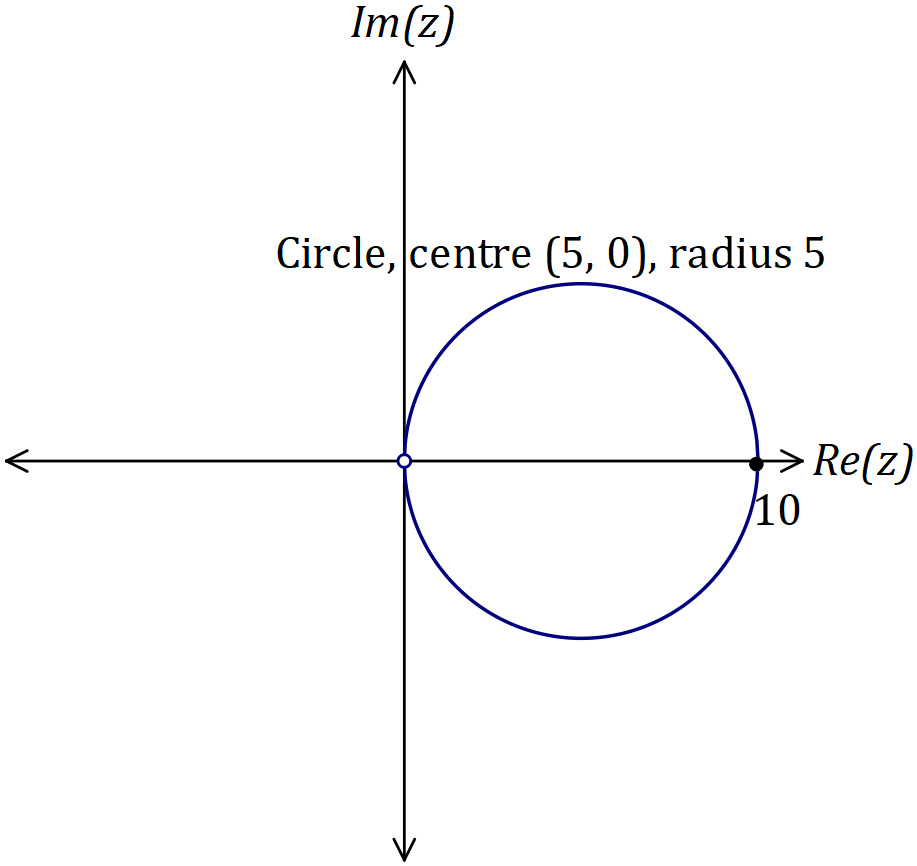
On the Argand planes below, sketch the locus of the complex number given by the following.

(a) . (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ plots 2 points   forms perpendicular bisector   indicates axes intercepts |

(b) . (4 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ multiplies equation by   simplifies, with   circle with correct centre and radius   excludes (0, 0) |

Question 1 (7 marks)

(a) Given that is a factor of , determine the values of the real constants and . (4 marks)

(b) Solve the equation . (3 marks)

Question 1 (7 marks)

(a) Given that is a factor of , determine the values of the real constants and . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ identifies root and substitutes  ✓ determines correctly  ✓ equates and parts  ✓ solves for constants |

(b) Solve the equation . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ factorises  ✓ solutions  ✓ solutions |

Question 4 (7 marks)

Let and .

(a) Determine the argument of . (3 marks)

(b) Determine the real part of . (4 marks)

Question 4 (7 marks)

Let and .

(a) Determine the argument of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates and  ✓ indicates addition of arguments  ✓ states required argument |

(b) Determine the real part of . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates  ✓ expresses in polar form  ✓ determines  ✓ states real part |

Question 8 (8 marks)

(a) Determine all solutions to the complex equation in the form where and . (5 marks)

(b) If is any complex cube root of unity, simplify . (3 marks)

Question 8 (8 marks)

(a) Determine all solutions to the complex equation in the form where and . (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses in polar form  ✓ uses De Moivre's theorem  ✓ expresses general solution in terms of  ✓ states one correct root  ✓ states all five roots correctly |

(b) If is any complex cube root of unity, simplify . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses cube of root  ✓ uses sum of roots  ✓ simplifies |